Non-parametric test for inter-subject correlations

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February 2, 2016

Abstract

This document describes the resampling test that ISC Toolbox (Kauppi et al., 2014) uses to make inferences about the intersubject correlations in mathematical terms. This document is focused only on the basic ISC analysis (in the terminology of Kauppi et al. (2014)) described in sections 2.2.1 and 2.2.2 of Kauppi et al. (2014). This document aims to fill the obvious need to have a mathematically oriented description of the procedure. The resampling test described here was introduced by Kauppi et al. (2010).

Versions:

7th December 2015: original version
2nd February 2016: Corrected a typo in the definition of $Res(\mathbf{x}_k(i), j)$; thanks to Melodie Angeletti for pointing that out

1 Introduction

Assume that we have fMRI time series recorded from K subjects during a natural stimulation experiment and all the data have been registered to a stereotaxic space. Denote the time series from subject k at the voxel i (in the stereotaxic space) by $\mathbf{x}_k(i) = [x_k(i,1), \ldots, x_k(i,N)]^T$. We consider each $\mathbf{x}_k(i)$ to be demeaned so that $\sum_{t=1}^{N} x_k(i,t) = 0$. To study the intersubject synchronization (or correlation), we first compute the correlation coefficient between subjects' time series

$$r_{kl}(i) = \frac{\mathbf{x}_k(i)^T \mathbf{x}_l(i)}{||\mathbf{x}_k(i)||\,||\mathbf{x}_l(i)||} \tag{1}$$

at each voxel i for each subject pair $\{k, l\}$. We use $R(i) = (r_{kl}(i))$ to denote the resulting $K \times K$ correlation matrix at voxel i. The basic statistical problem is then to test if there is significant synchronization in neuronal activity at the voxel i between the subjects using $R(i)$. Note that this does not necessarily account of testing null-hypothesis of zero correlation because BOLD fMRI is an indirect measure of neuronal activity, and there are various imaging artifacts which may confound the hypothesis testing. Other complications include 1) temporal correlations in fMRI time series, 2) dependencies between the elements of the matrix $R(i)$, and 3) the spatial dependencies between $R(i)$ and $R(j)$ for the nearby voxels i and j; in the massively univariate setting this becomes an issue during the correction for multiple comparisons.

2 Inference against null hypothesis of no synchronization

Researchers have used different strategies to make inferences about the correlation matrices $R(i)$. We have adopted a resampling based procedure similar to the verification procedure of Wilson et al. (2008) for the ISC based testing for BOLD synchronization. In this setting, we consider the mean of the correlation coefficients for a single voxel as the statistic

$$S(\mathbf{x}_1(i), \ldots, \mathbf{x}_K(i)) = \frac{\sum_{k=1}^{l-1} \sum_{l=1}^{K} r_{kl}(i)}{\hat{K}} = (1/\hat{K}) \sum_{k=1}^{l-1} \sum_{l=1}^{K} \frac{\mathbf{x}_k(i)^T \mathbf{x}_l(i)}{||\mathbf{x}_k(i)|| \, ||\mathbf{x}_l(i)||},$$

where $\hat{K} = K(K-1)/2$ be the number of subject pairs. The distribution for S under the null hypothesis of no-synchrony is then constructed by resampling time series $\mathbf{x}_k(i)$ by circularly shifting them as follows. Define circularly j-shifted time series as

$$Res(\mathbf{x}_k(i), j) = [x_k(i, 1+j), x_k(i, 2+j), \ldots, x_k(i, N), x_k(i, 1), \ldots, x_k(i, j)]^T,$$

where $j \geq 0$ is an integer. Now we can define the resampling procedure.

For A times $a = 1, \ldots, A$, draw a random voxel i and K integers $j_1, \ldots, j_K$ with $0 \leq j_b \leq N-1$ for every $b = 1, \ldots, K$. Compute

$$S_a = S(Res(\mathbf{x}_1(i), j_1), \ldots, Res(\mathbf{x}_K(i), j_K));$$

The null distribution for S is $\{S_a : a = 1, \ldots, A\}$.

Several points can be made about the procedure: 1) In (Kauppi et al., 2010) (but not in Kauppi et al. (2014)) resampled also over the frequency band. However, now there is only one frequency band (whole band) and we drop the consideration of frequency bands to avoid clutter in notation. 2) The re-sampling process shares high degree of similarity with circular block bootstrap (Politis and Romano, 1992; Lahiri, 1999). 3) The temporal characteristics of the original time series are preserved by the resampling procedure. 4) The simultaneous timing between subjects is broken, which is exactly what we wanted. The resampling procedure simulates the scenario where the subjects start to experience the stimulus from a randomized point of the stimulus. 5) Note that resampling does not alter the denominator of (1), so it does not have to be re-computed and also that the resampled time series has the same mean as the original one.

Multiple comparisons correction was performed with false discovery rate criterion using the standard Benjamini-Hochberg procedure (Benjamimi and Hochberg, 1995), mainly due to inadequacy of the extreme statistics based permutation approach for the current problem.

References

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